Stopping Criteria for Value Iteration on Stochastic Games with Quantitative Objectives

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Talk in one slide

- **Probabilistic systems**: Best algorithm (usually) is **Value Iteration (VI)**
- But: Requires a **stopping criterion**
  For **Stochastic Games (SG)** with most **infinite-horizon, quantitative objectives** there is **none!**

- This paper: **Uniform** solution for **large class of quantitative objectives** (including total reward, mean payoff, …)

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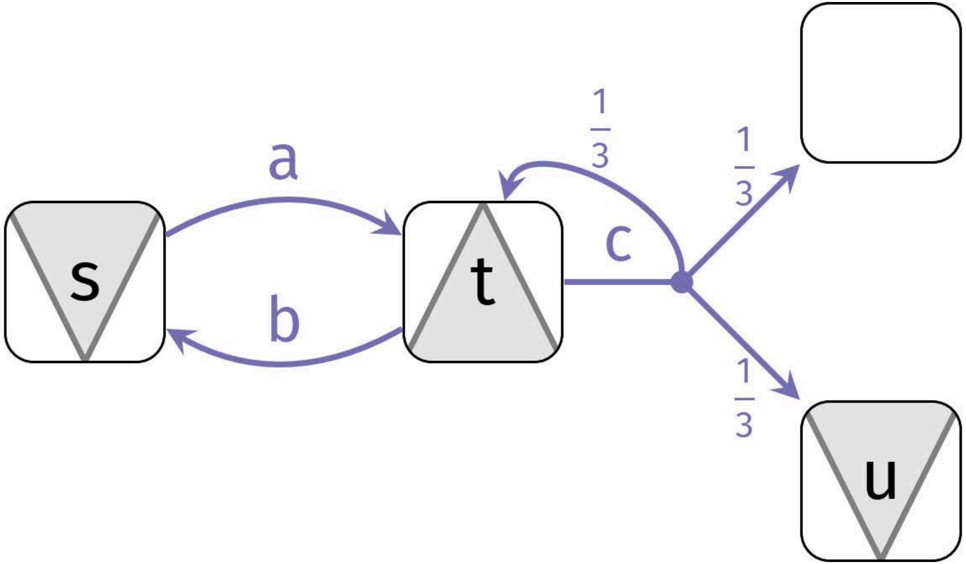
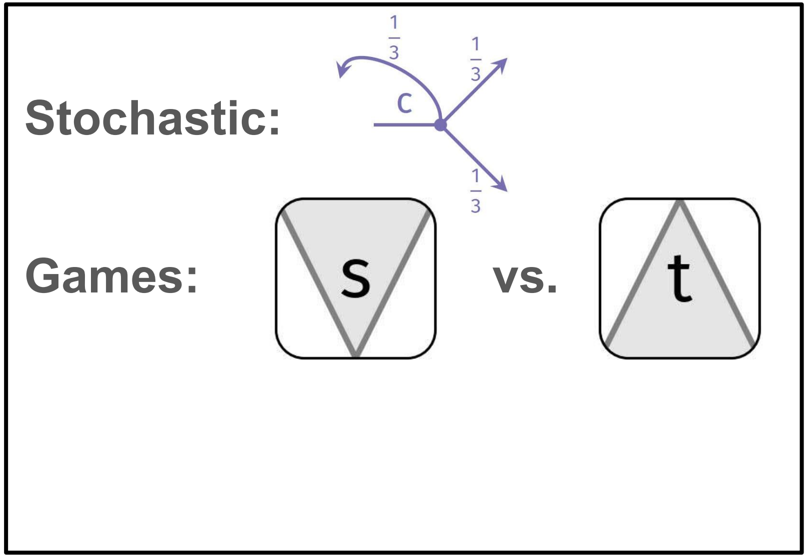
- This paper: **Uniform** solution for **large class of quantitative objectives** (including total reward, mean payoff, …)

Unifies all previous ones and is more broadly applicable.

Stochastic Games





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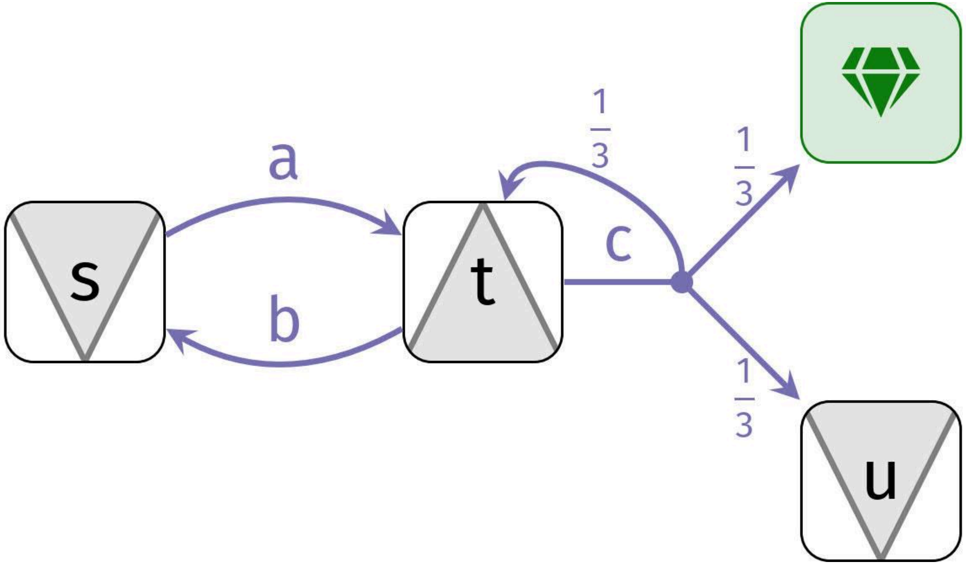
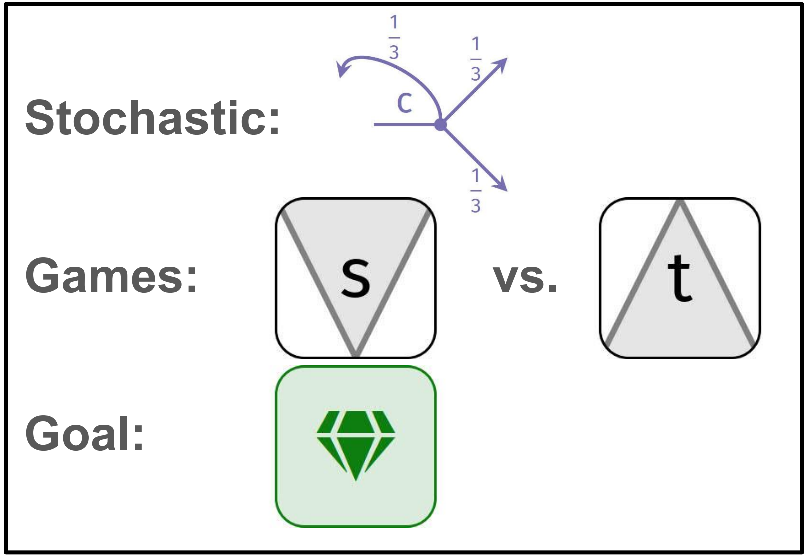
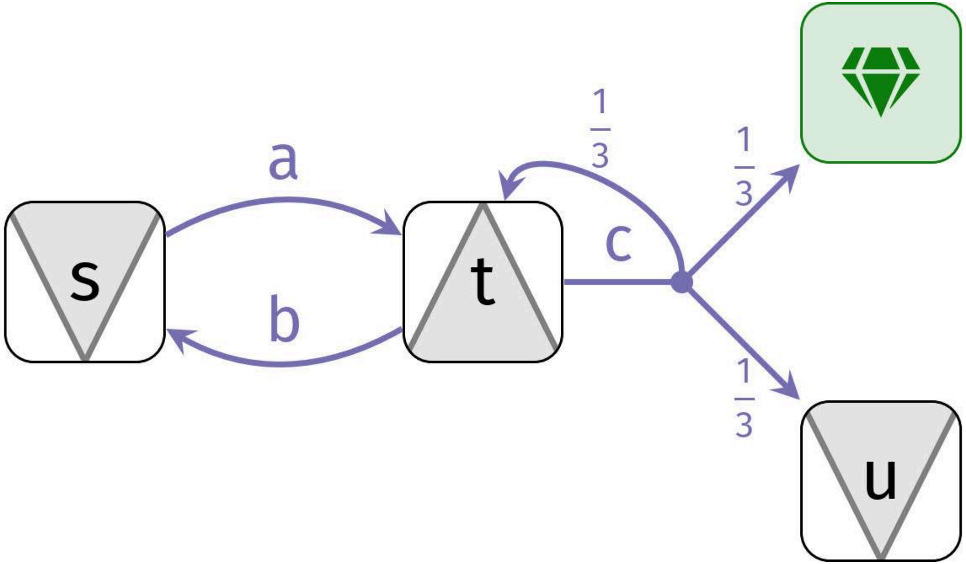


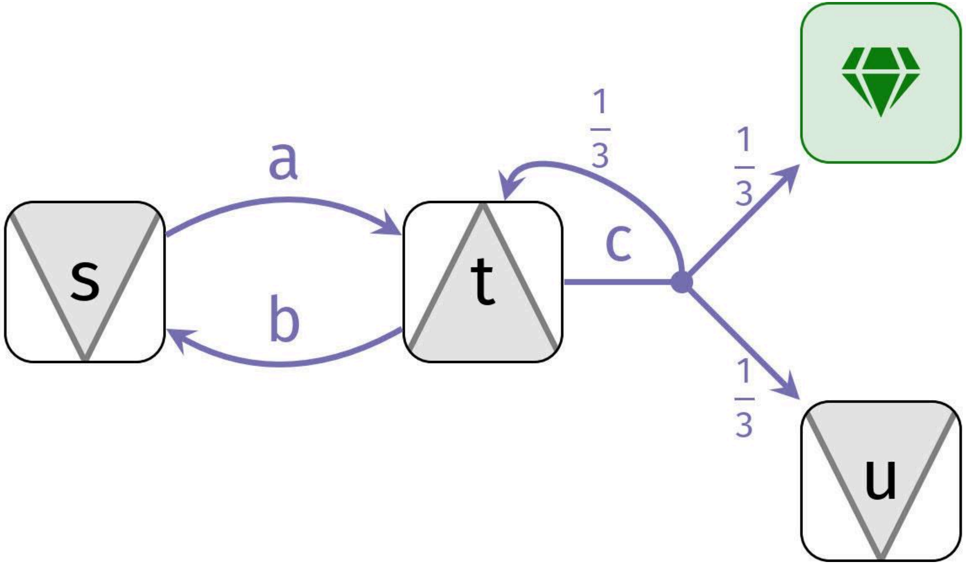


Stochastic Games and Value Iteration



Iteration	L(s)	L(t)
0	0	0
1		
2		
…		

Stochastic Games and Value Iteration



Bellman update:

$$x_i(s) = \mathbf{opt}_a \; x_{i-1}(s,a)$$

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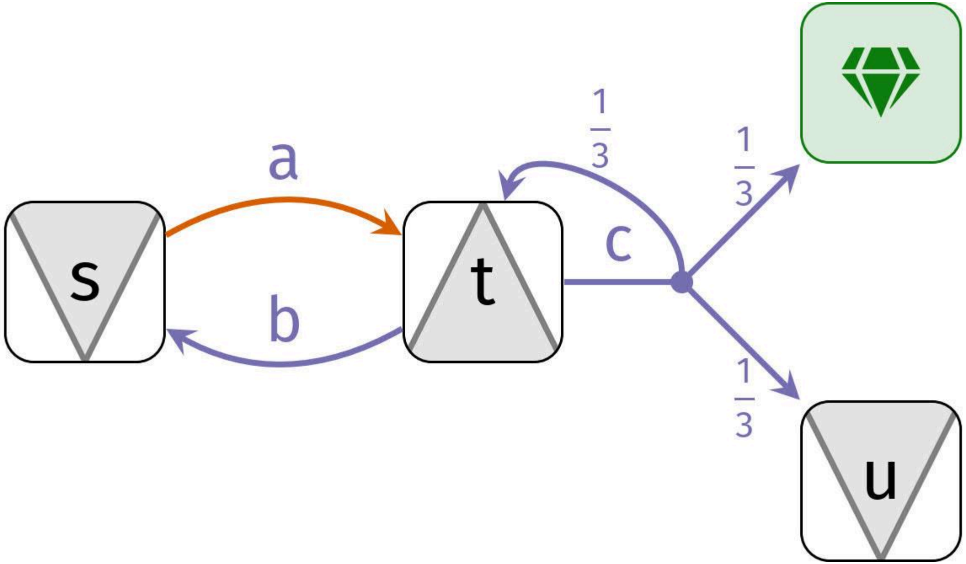

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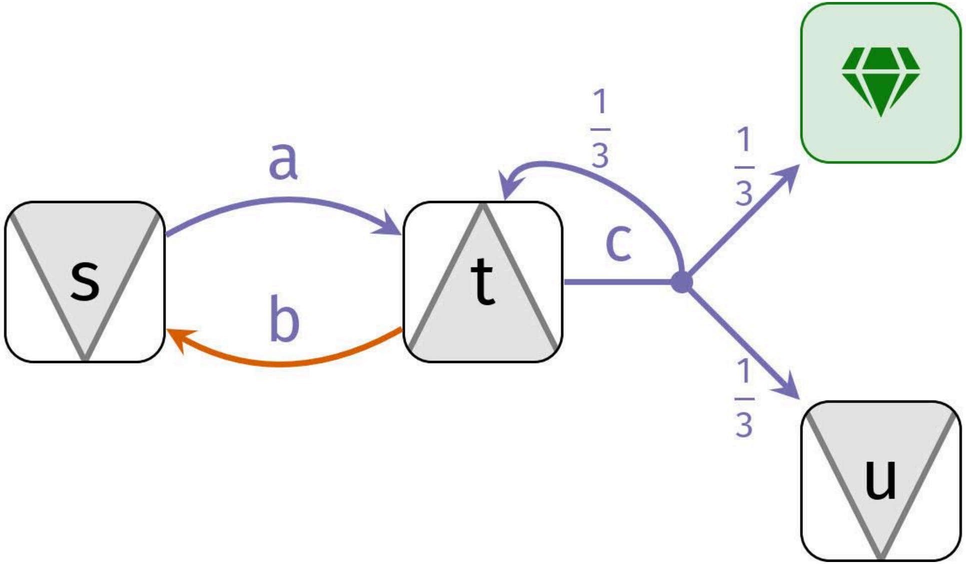

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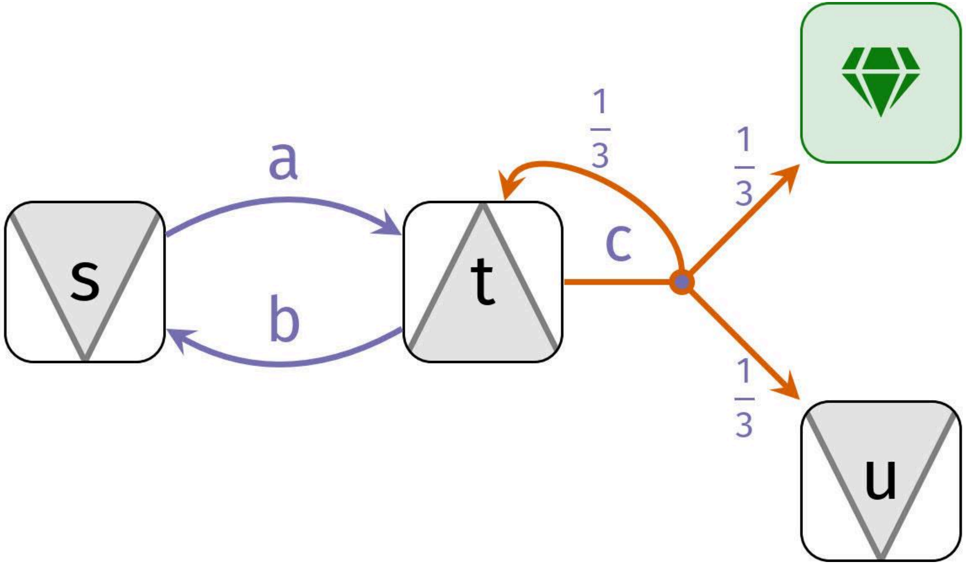

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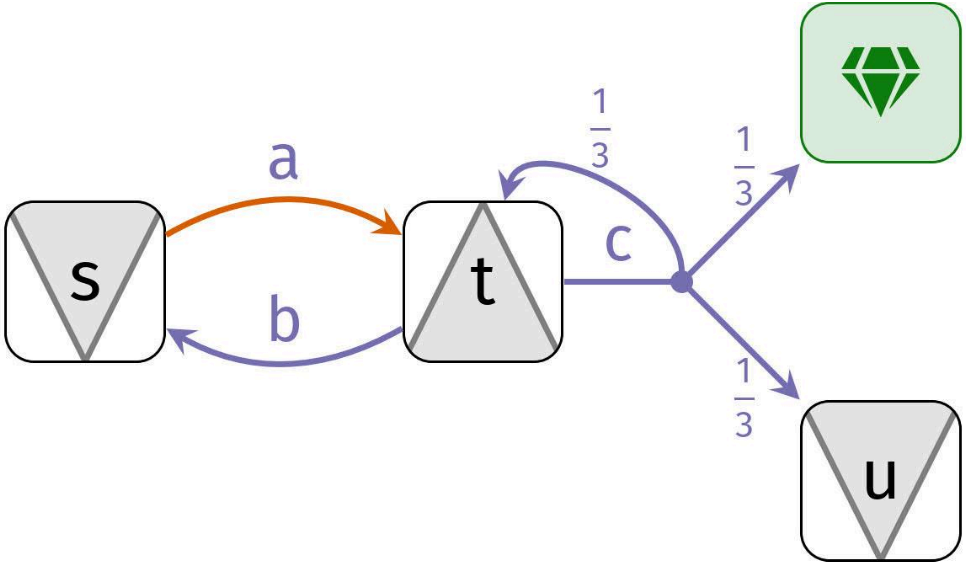

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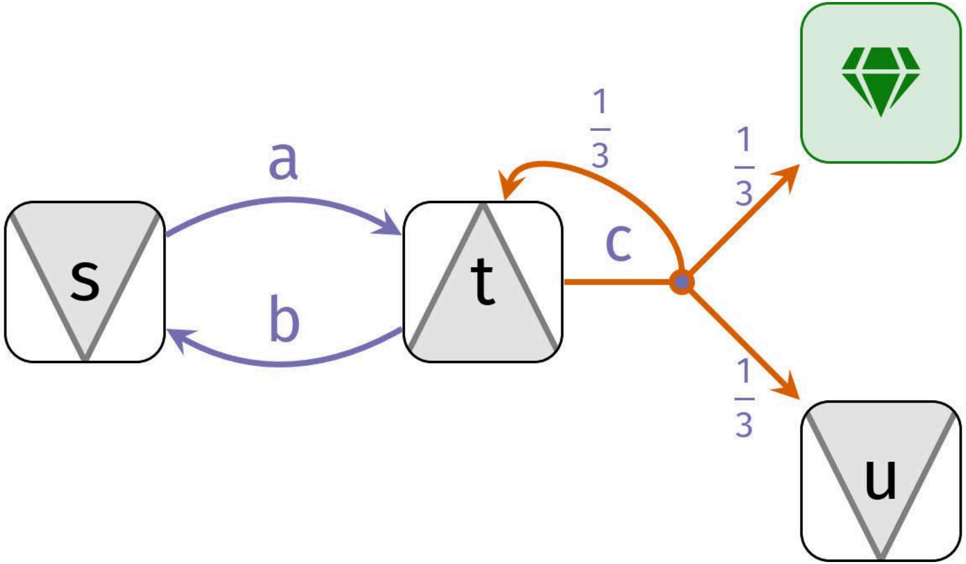

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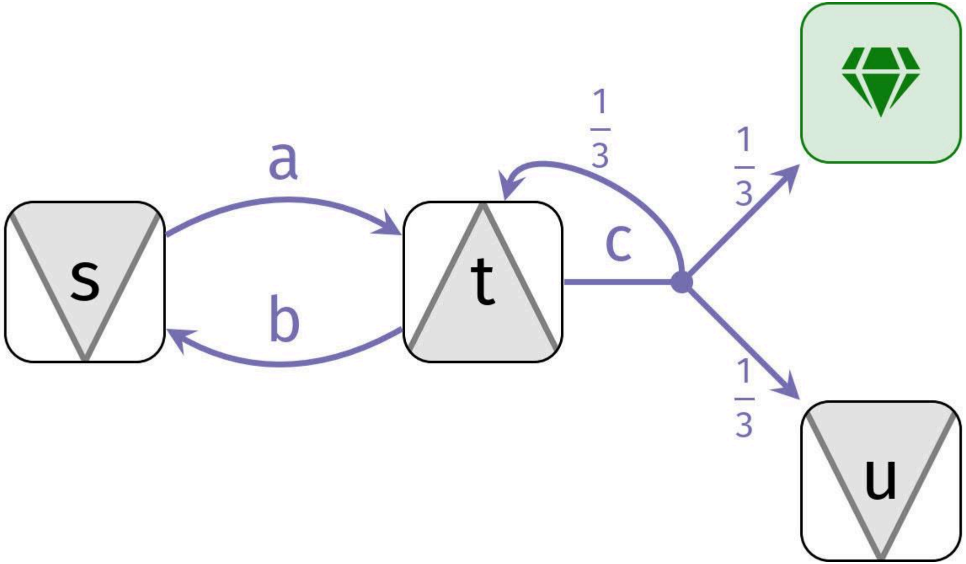

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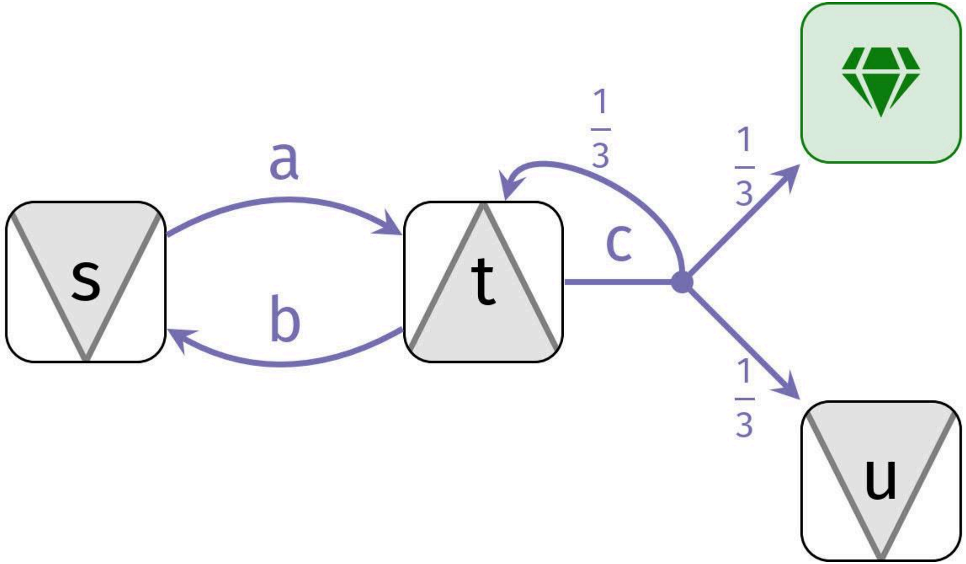

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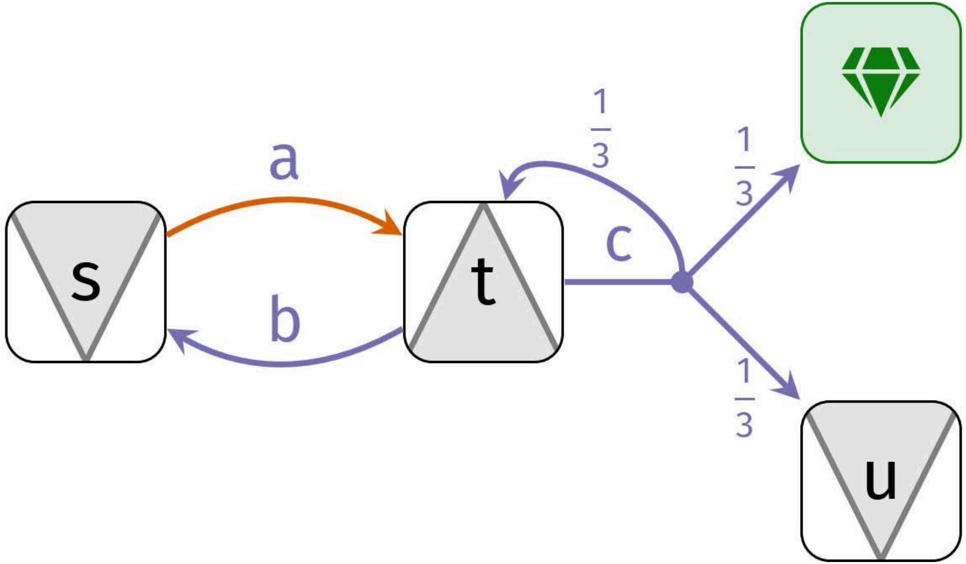

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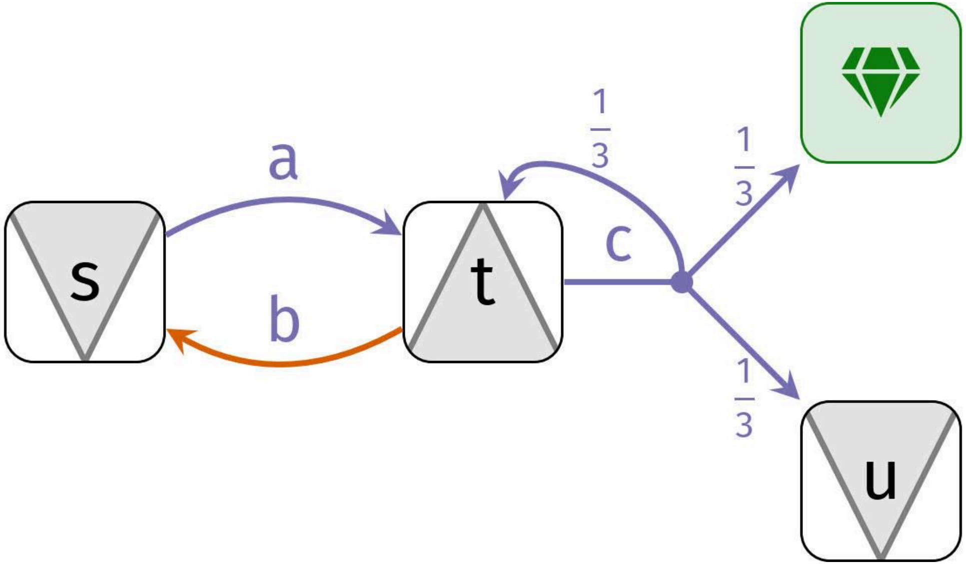

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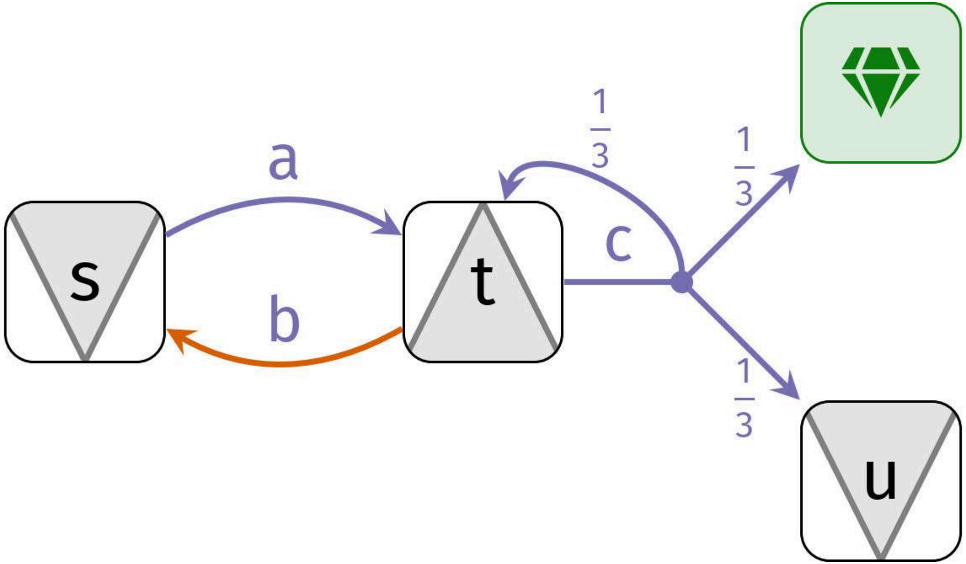

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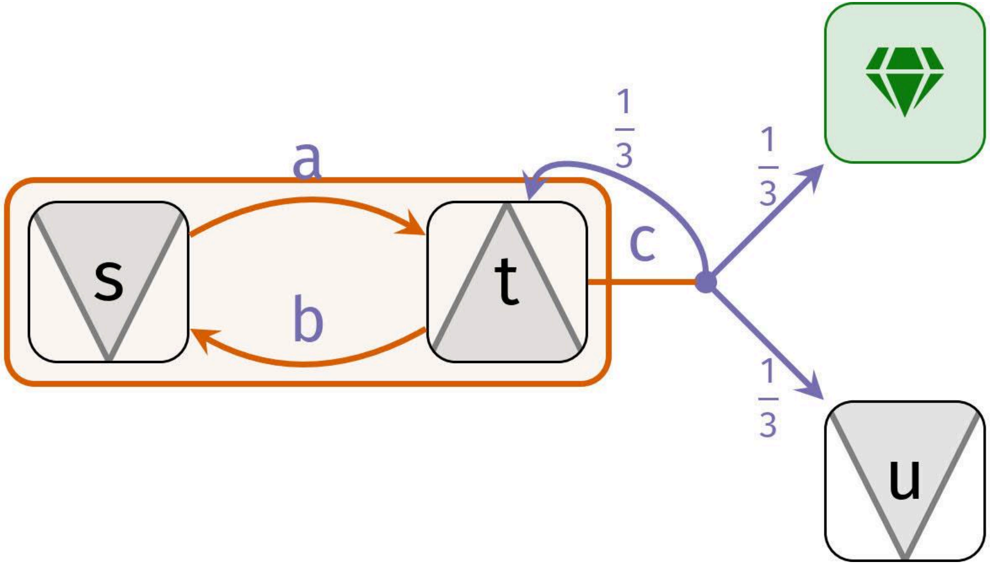

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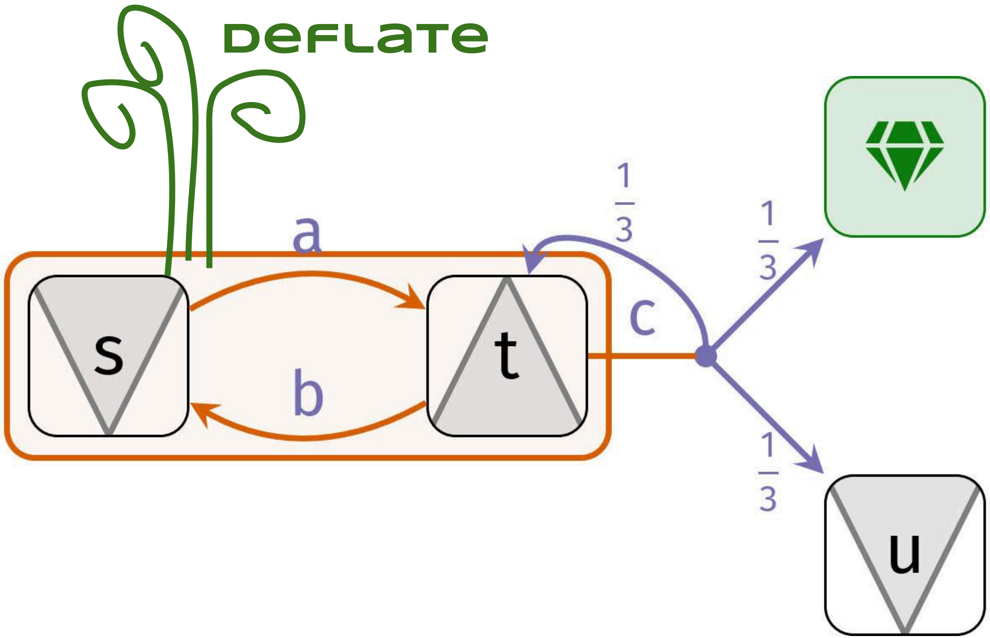

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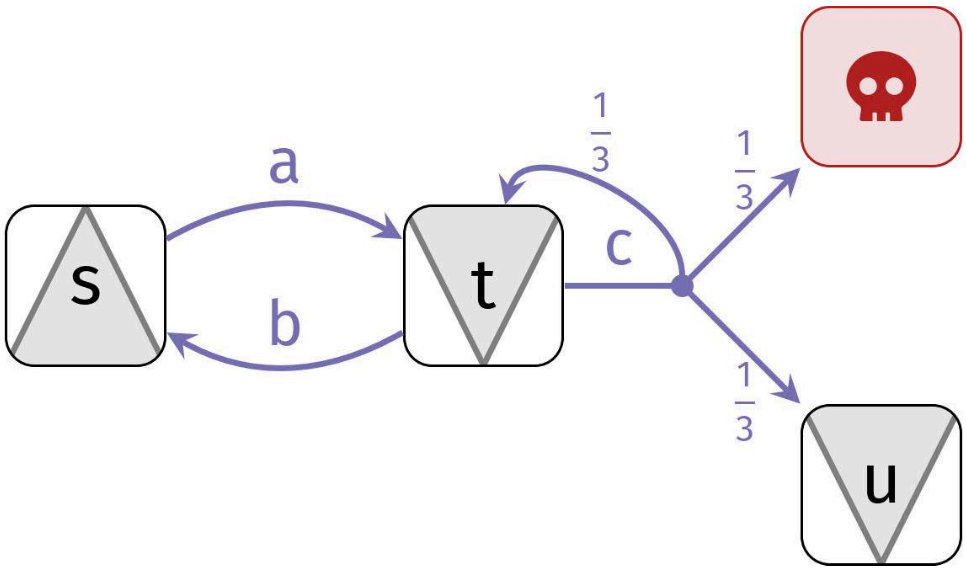
1	0	1/3		1	**2/3**
2	1/3	4/9		**2/3**	**5/9**
…	…	…		…	…

Stochastic Games and Value Iteration



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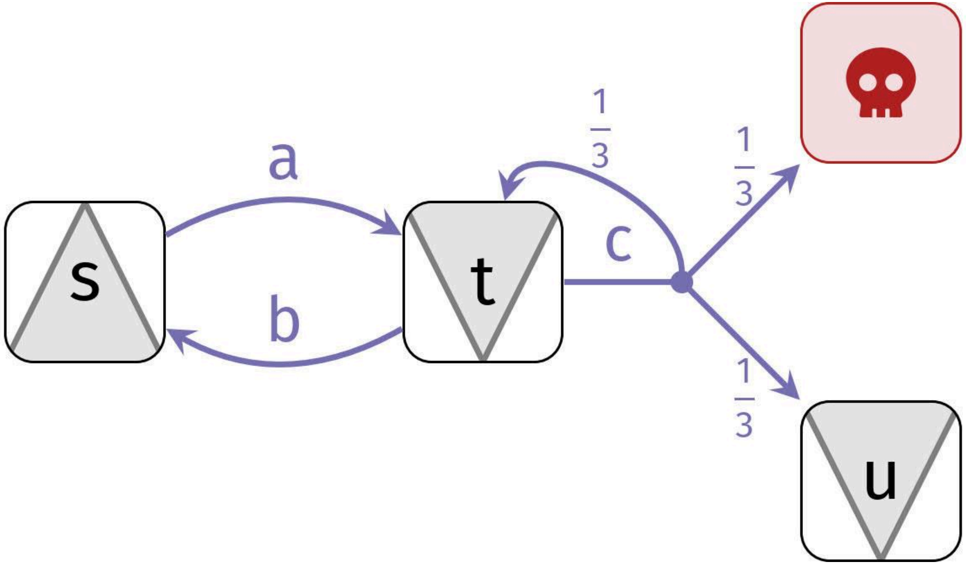

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0	0	0	1
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2			
…	…	…	

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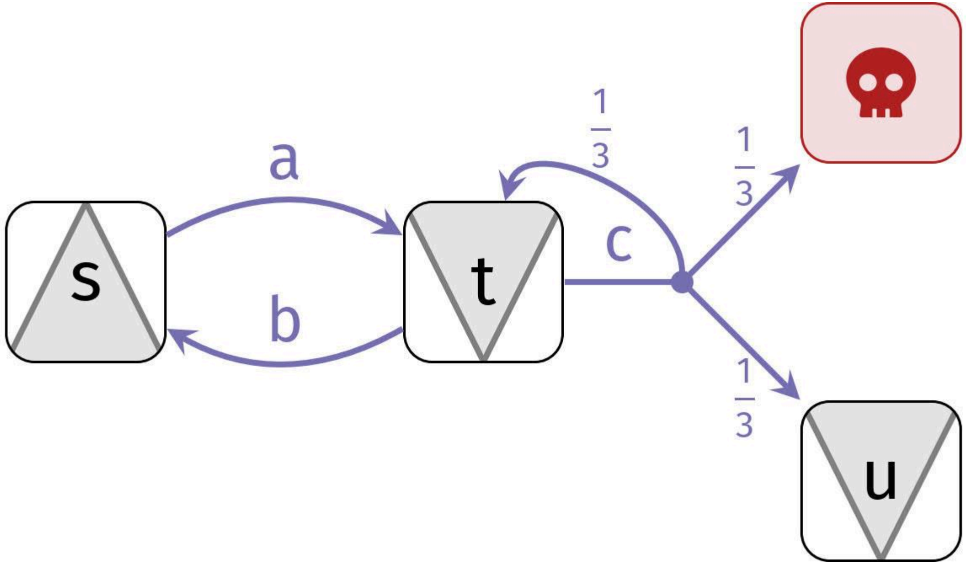

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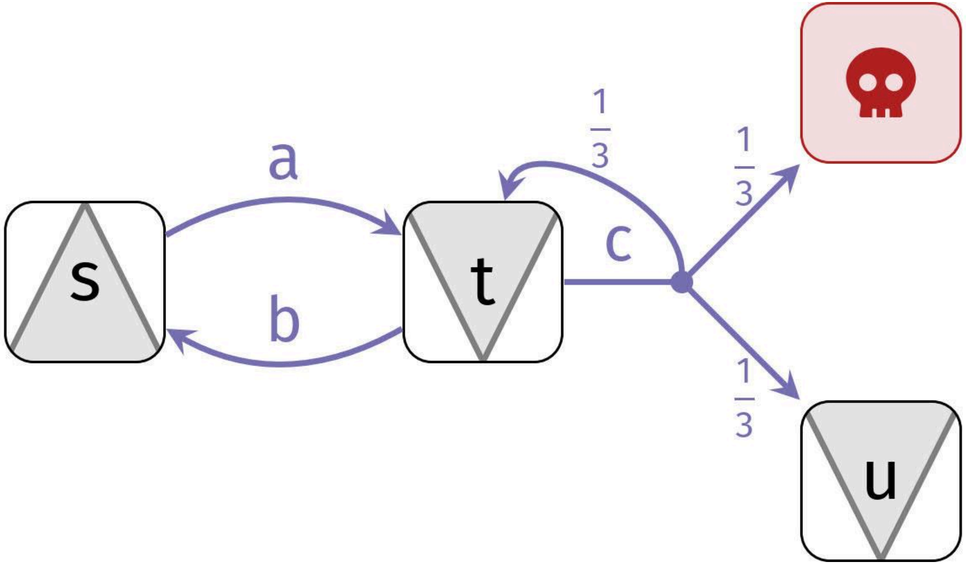

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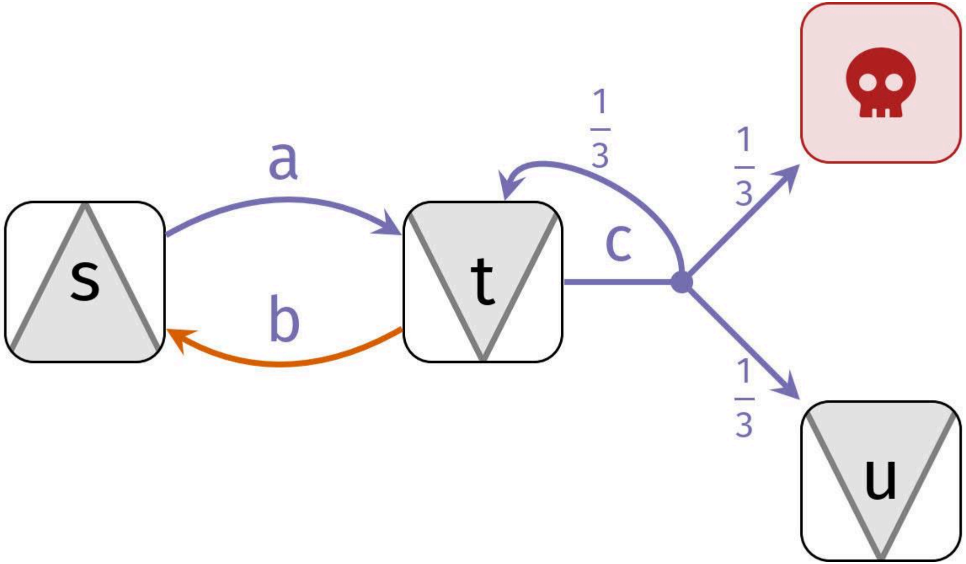

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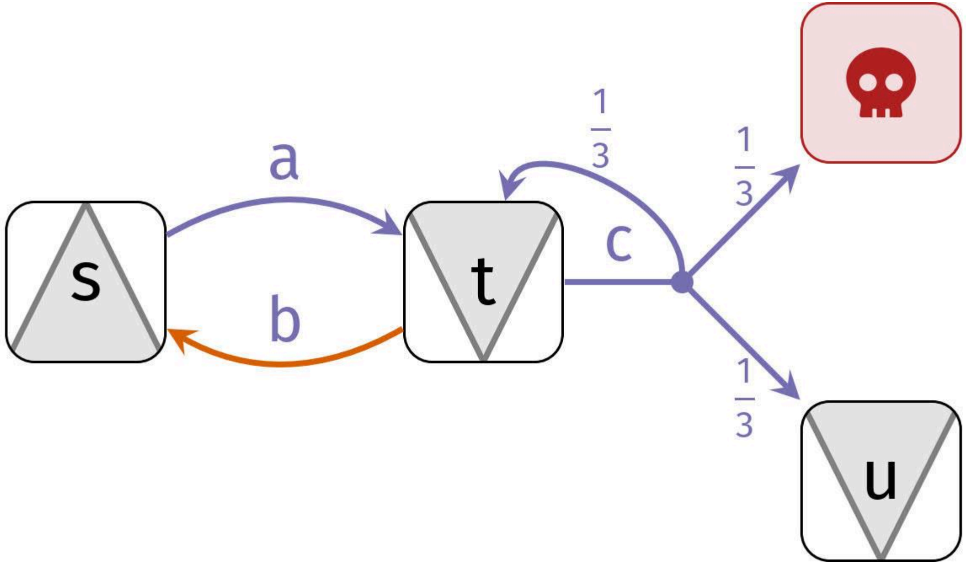

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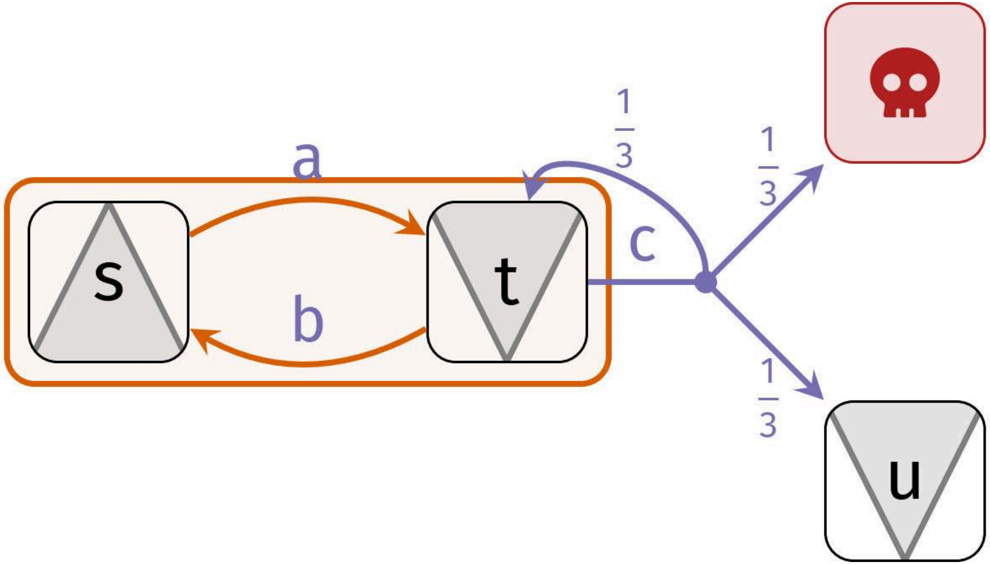

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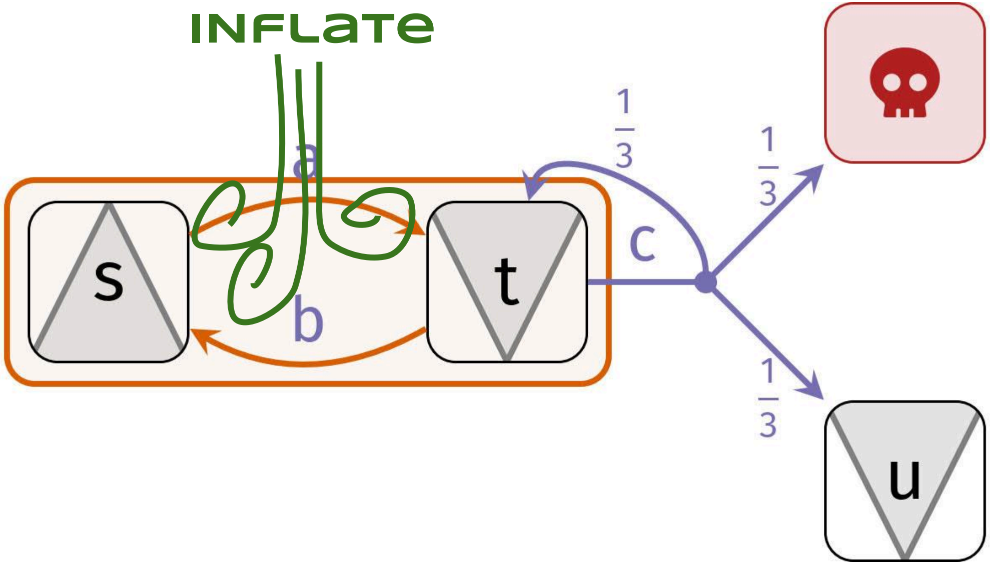

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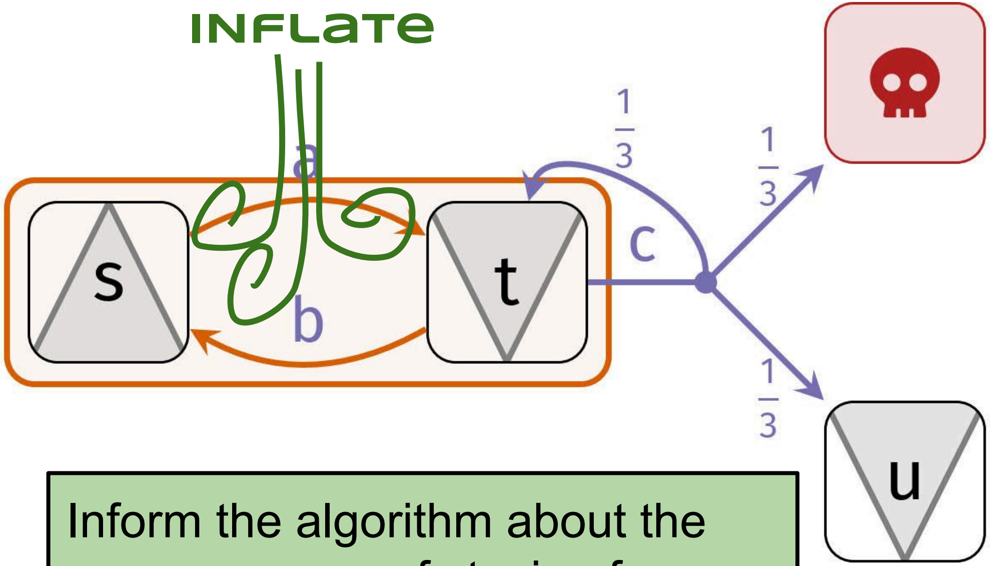
Stochastic Games and Value Iteration



Inform the algorithm about the consequences of staying forever.

Bellman update:

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Should I stay or should I go now?

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Reachability:

stay=0

Safety:

stay=1

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stay=0 max(stay,exit) = exit

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Reachability:

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min(stay,exit) = exit

Should I stay or should I go now?

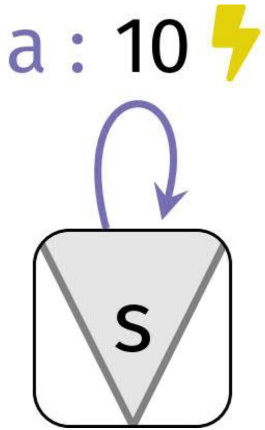
Reachability:

stay=0	max(stay,exit) = exit
min(stay,exit) = 0

Safety:

stay=1	max(stay,exit) = 1
min(stay,exit) = exit

Deflating and Inflating for Mean Payoff



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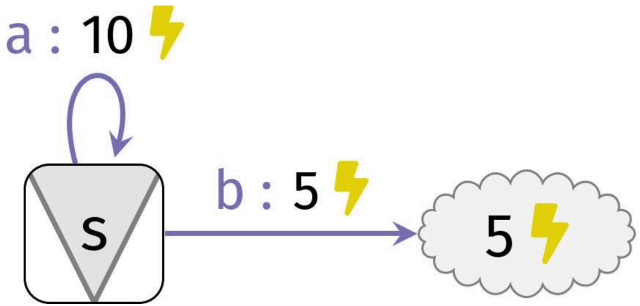

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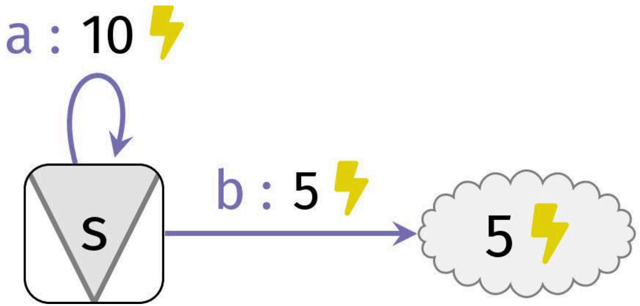

Inflate from 0 to **exit** value 5

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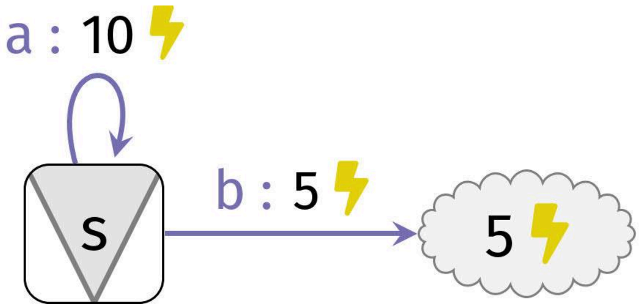

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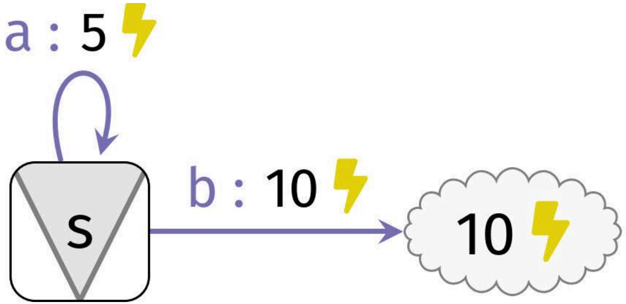

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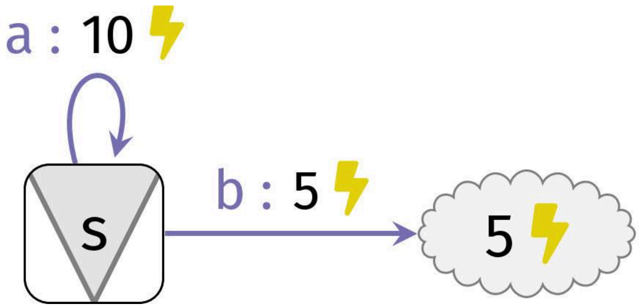

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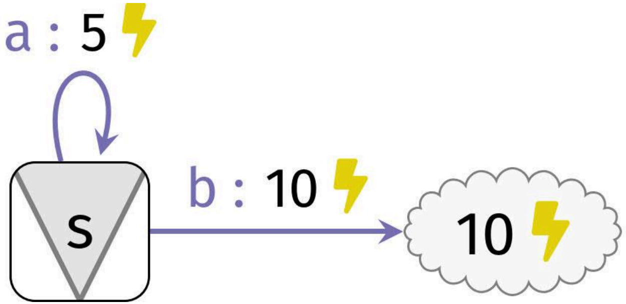
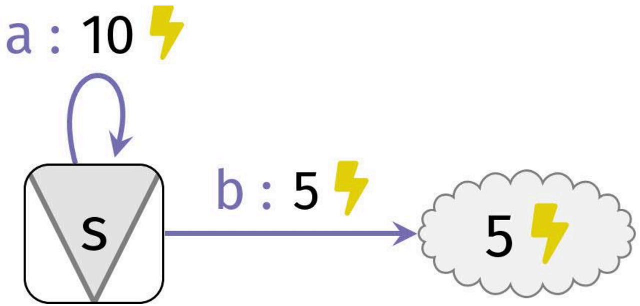


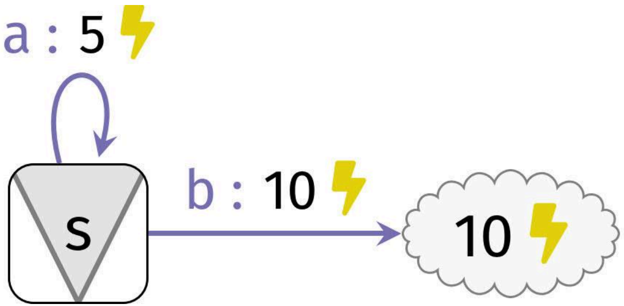
Inflate from 0 to **stay** value 5

Deflating and Inflating for Mean Payoff



Inflate from 0 to **exit** value 5

And dually for Maximizer states



Inflate from 0 to **stay** value 5

Where can I stay and go?

- Idea: If opponent thinks staying here is good,
  what happens if we really do stay?

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- Thus: Fix opponent’s strategy and analyse remaining cycles.

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SOUND

COMPLETE

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SOUND

COMPLETE

OBJECTIVE-INDEPENDENT

Conclusion

- Given: **Stochastic Games** with **quantitative objectives** (including reachability, safety, mean payoff, expected total reward, …),
- Goal: Solving them **quickly** and with **precision-guarantees**
- Approach: **Value Iteration** with our **new stopping criterion**

Idea: Inform the algorithm about the consequences of staying forever:
Should I stay or should I go now?

Unifies previous work [BCC+14, HM14, BKL+17, ACD+17, KKKW18, PTHH20]
in an **elegant, objective-independent way**